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Nonequilibrium Phenomena: Outlines and Bibliographies of a Workshop

P. C. Hohenberg[1,2] **and J. S. Langer**[1,3]

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The following is a set of outlines and bibliographies for lectures presented at a Summer Workshop on Nonequilibrium Phenomena held from June 22 to July 3, 1981 at the Institute for Theoretical Physics in Santa Barbara. These outlines were distributed to the participants in lieu of formal proceedings, and they are being presented for publication in the same form, in the belief that the information they contain will be useful to a wider audience. It should be clearly stated, however, that the compilation is an informal one which does not claim to be a complete survey of the subject.

KEY WORDS: Nonequilibrium phenomena; dynamical systems; hydrodynamic stability; onset of turbulence.

A. SUMMARY OF THE WORKSHOP

The main purpose of this workshop was to examine recent mathematical developments in the areas of nonlinear systems, bifurcation theory, and ergodicity, and to explore applications of these developments to physical theories of nonequilibrium phenomena. The phenomena of principal interest were instabilities, pattern formation, and the transition from regular to chaotic behavior. The physical situations included convection, Taylor–Couette flow, nucleation, spinodal decomposition, solidification, chemical reactions, and biological processes.

The first goal of the lectures and discussions was to identify important questions in the field of nonequilibrium phenomena. Among these questions were the following:

[1] Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
[2] Bell Laboratories, Murray Hill, N.J. 07974
[3] Dept. of Physics, CMU, Pittsburgh, PA. 15213.

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How does one characterize and classify the steady states which are obtained under constant external conditions away from thermodynamic equilibrium? [It is assumed that these steady states have simpler behavior than arbitrary nonequilibrium states.]

Are there general techniques for classifying the spatial and temporal patterns which emerge in such systems?

How are patterns selected? Under what conditions, if any, is there a functional whose extrema describe steady states? What is the role of noise in pattern selection? What is the role of defects in spatial structures?

What are the essential differences between externally driven nonequilibrium steady states, e.g., cellular convection, Taylor–Couette flow, etc., and freely equilibrating systems such as those undergoing nucleation or spinodal decomposition?

Is there a deep similarity between thermodynamic phase transitions and transitions between steady states of nonequilibrium systems, e.g., transitions from thermal conduction to stationary convection, from stationary convection to oscillation, or from oscillation to temporal chaos, in the Rayleigh–Bénard system? Do the latter transitions exhibit any form of universality?

What are the relative merits of various theoretical methods for the study of nonequilibrium phenomena, e.g., rigorous mathematical theorems, exactly soluble models, systematic expansions, phenomenological theories, computer simulations?

What physical systems reveal basic nonequilibrium phenomena most simply? Conversely, how do we interpret various natural phenomena in terms of basic principles? Pattern formation and chaotic behavior occur in geophysics, oceanography, astrophysics, ecology, biology, chemistry, metallurgy, etc. What insight do we gain concerning natural phenomena by the study of simple experimental systems and mathematical models?

This program of lectures is summarized in the list of speakers and topics and described in more detail in the accompanying outlines and bibliographies.

The series of lectures by Lanford, Marsden, Rand, and Guckenheimer provided mathematical background in the theory of dynamical systems with emphasis on bifurcation theory and the use of measure theoretic concepts. These lectures dealt in some detail with the classification of steady states in relatively simple systems—the occurrence of fixed points and limit cycles and the mechanisms by which such structures can evolve into much more complicated ones containing "strange attractors," "strange saddles," etc.

Much attention was paid to the use of one- or two-dimensional maps as representations or models of dynamical systems. Feigenbaum described the appearance of universal (model-independent) features in one-dimensional maps which exhibit cascades of period-doubling transitions. The recent observation of such transitions in Rayleigh–Bénard convection offers the possibility of applying the Feigenbaum theory to real systems (see lectures by Ahlers).

A topic which arose throughout the workshop was the role of noise, both imposed by external sources and arising dynamically within the system itself. Methods for dealing with systems driven by external noise were described by Caroli in her lectures on stochastic processes. The problems of characterizing the noise spectrum in deterministic systems exhibiting chaos were discussed by Lanford and Rand (lectures VI and XII). A new and important class of problems arises when one imposes noise externally on a deterministic system which, by itself, undergoes a transition to chaotic behavior. These problems were described by Rudnick and also touched upon by Rand (XII) and by Martin in his concluding remarks.

The most carefully studied physical realizations of nonequilibrium phenomena occur in hydrodynamics, especially Rayleigh–Bénard convection and Taylor–Couette flow. The experimental situation in this area was described in the lectures by Gollub, Ahlers, Swinney, and Busse. Related theoretical developments were presented by Cross, Busse, Rand, and Siggia. Work in this area falls roughly into two categories: systems with a small number of convective cells where only a small number of hydrodynamic degrees of freedom seem to be relevant, and large systems with many degrees of freedom where the picture changes qualitatively. In the case of small systems which are stabilized by lateral boundaries, there are indications that the various routes from conduction through convection to chaos may be found to correspond to generic behavior of a relatively small class of tractable dynamical models. For large systems, on the other hand, problems relating to pattern selection and stability become acute, and no clear picture has yet emerged.

The remaining lectures in the workshop were devoted to physical systems where nonhydrodynamic features come into play, but where many of the concepts discussed above ought still to be important. Gunton's lectures on the kinetics of first-order phase transitions pertained explicitly to equilibrating as opposed to steadily driven systems—systems for which a free-energy minimization principle clearly exists and, at least in principle, solves the pattern-selection problem. Langer discussed pattern formation during solidification, also a first-order phase transition, but one in which the interesting phenomena are best described in the language of dynamical systems. In particular, directional solidification problems bear analogies to Rayleigh–Bénard convection in large systems, and pose essentially the same questions regarding pattern selection. Dendritic solidification seems to be qualitatively different, and poses a number of interesting new stability problems. The dynamics of chemical reactions were discussed by Swinney in a specific example of a system exhibiting periodic and chaotic behavior, and also by Ortoleva in a variety of chemical and geological systems. Finally, Ortoleva and Segel presented a number of examples of complex dynamical behavior in biological and ecological systems, where one might hope to find natural examples of the phenomena studied earlier.

B. LIST OF SPEAKERS AND TOPICS

P. Hohenberg:	Introductory Overview
J. Guckenheimer, O. Lanford, J. Marsden, D. Rand	Mathematical Theory of Dynamical Systems
M. Feigenbaum:	Universality in Period-Doubling Models
J. Rudnick:	Period Doubling in Iterated Maps
J. Gollub:	Studies of the Transition to Turbulent Convection Using Scattered Laser Light
G. Ahlers:	Experiments on Horizontal Layers of Fluid Heated from Below
H. Swinney:	Instabilities and Chaos in the Couette–Taylor System and Complex Dynamics in Nonequilibrium Chemical Reactions
M. Cross:	Amplitude Equation for the Description of Convection near Onset
F. Busse:	Hydrodynamics of Convection
E. Siggia:	Topics in the Theory of Convective Structures
J. Gunton:	Kinetics of First-Order Phase Transitions
C. Caroli:	Introduction to Stochastic Processes
J. Langer:	Solidification Patterns
P. Ortoleva:	Nonequilibrium Phenomena in Chemistry and Biology
L. Segel:	Nonequilibrium Phenomena in Biology and Ecology
P. Martin:	Concluding Summary

C. OUTLINES AND BIBLIOGRAPHIES

Mathematical Theory of Dynamical Systems

J. Guckenheimer
Department of Mathematics

University of California
Santa Cruz, California 95064

J. Marsden and O. Lanford
Department of Mathematics
University of California
Berkeley, California 94720

D. Rand
Mathematics Institute
University of Warwick
Coventry, England

I. Introduction to Dynamical Systems (Marsden)
1. Basic terminology, flows, fixed points, characteristic exponents, periodic orbits, examples
2. Invariant manifolds, insets, outsets, homoclinic orbits
3. Poincaré maps, forced oscillations, periodic orbits
4. Bifurcations, saddle-node, Hopf, period doubling, torus, homoclinic bifurcations

(Refs. 56, 28, 7, 1, 2)

II. Invariant Measures and Ergodic Theory (Lanford)
1. Existence of time averages is not automatic
2. Statistically regular orbits and their asymptotic distributions
3. Terminology: measure, probability measure, invariance of a measure under a mapping, abstract dynamical system
4. Poincaré recurrence theorem
5. Birkhoff pointwise ergodic theorem

} and their limitations: set of measure zero need not be small

6. Ergodicity and mixing property: definition and interpretation
7. Examples

(Refs. 24, 8, 55)

III. The Horseshoe and Solenoid (Rand)
1. Relation to homoclinic orbits
2. Geometric construction of the horseshoe, invariant set Λ, nonwandering points
3. Symbolic dynamics, periodic and dense orbits, Cantor set
4. Solenoid as a strange attractor, horseshoe as a strange saddle

(Refs. 56, 48, 9)

IV. Ruelle–Bowen Ergodic Theorem (Lanford)
   1. Rough statement
   2. Sketch of proof for the solenoid

   (Refs. 9, 10, 53)

V. Finding Horseshoes (Marsden)
   1. Discussion of why horseshoes may be important, comparison with strange attractors
   2. Horseshoes and other bifurcations in forced oscillations; Duffing and beam equations
   3. Horseshoes in autonomous equations, pendulum-oscillator and rigid body

   (Refs. 56, 30–36)

VI. Lyapunov Exponents and Elementary Time Series (Lanford)
   1. Lyapunov exponents
      a. One-dimensional case (and why the multidimensional case is essentially more complicated)
      b. Oseledec multiplicative ergodic theorem; definition of Lyapunov exponents

   (Ref. 54)

   2. Time series (elementary)
      a. Definitions: covariance, periodogram, power spectrum
      b. Relation between periodogram and power spectrum
      c. Why are measured power spectra "rough" and how can they be smoothed?

   (Ref. 11)

VII. Modulated Waves (Rand)
   1. Axisymmetric dissipative systems and the route to turbulence via rotating and modulated waves
   2. Spatiotemporal symmetry group
   3. Stationary axisymmetric flow
   4. Onset of time dependence
   5. Rotating waves
   6. Bifurcation to quasiperiodic flow
   7. Modulated waves
      a. Phase functions and wave frequencies
      b. Spatiotemporal structure

c. Dynamics and frequencies
d. Possible values of the modulation angle

(Refs. 52, 16)

VIII. Rigorous Results on the Feigenbaum Cascade (Lanford)
1. The one-dimensional doubling operator
2. Status of proofs of the Feigenbaum conjectures
3. Construction of the n-dimensional doubling operator
4. The Feigenbaum conjectures in n dimensions follow from the one-dimensional case

(Refs. 15, 14, 39, 40)

IX. Codimension One Bifurcations (Guckenheimer)
1. Introduction via the Rayleigh–Bénard convection problem
2. Normal forms for saddle-node, Hopf, and pitchfork
3. The role of the trivial solution and symmetry in the pitchfork
4. Saddle-loop bifurcation

X. Bifurcation Theory for Partial Differential Equations (Marsden)
1. Center manifolds for evolution operators
2. Use of center manifolds in bifurcation problems
3. Reduction to finite dimensions
4. Examples: Navier–Stokes equations, reaction–diffusion equations, panel flutter

(Refs. 44, 25, 12, 27)

XI. The Lorenz Attractor (Guckenheimer)
1. Geometric description of Lorenz attractor
2. Lorenz equations
3. Bifurcation sequence ($0 < R < 25$) producing Lorenz attractor interpreted with one-dimensional mapping

(Refs. 42, 38, 22)

XII. Reconstructing Dynamical Systems (Rand)
1. Reconstructing an attractor from a single time series
2. Determination from the time series of metric quantities such as the capacity of the attractor, topological entropy, asymptotic measure, and metric entropy

(Refs. 57, 52)

XIII. Codimension-Two Bifurcations (Guckenheimer)
1. Rotating and double diffusive convection as motivating examples
2. Codimension-two bifurcations as technique for finding sequence of bifurcations
3. Classification and analysis of codimension-two bifurcations
4. Applications of theory to rotating convection

(Refs. 19, 20)

XIV. Horseshoes and Arnold Diffusion (Marsden)
1. Horseshoes for the forced Duffing equation
2. Horseshoes in Hamiltonian systems
3. Arnold diffusion
4. KAM theory compatible with Melnikov method

(Refs. 46, 4, 1, 6, 26, 31–36)

XV. Strange Attractors (Guckenheimer)
1. Axiom A attractors, and lack of axiom A attractors in physical systems
2. Henon "attractor" as model problem
3. Newhouse theorem
4. One-dimensional map as singular limit of Henon map
5. Theory of rotation numbers
6. Jacobson's theorem

(Refs. 49, 50, 37, 18)

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Universality in Period-Doubling Models

Mitchell J. Feigenbaum
Los Alamos National Laboratory
Los Alamos, New Mexico 37545

1. An intuitive account of the fixed point theory
   a. Rate of parameter convergence
   b. Local scaling
   c. Universality as a consequence of the fixed point of an operator
2. The formal fixed point theory
   a. The fixed point functional equation
   b. The derivative map at the fixed point and its spectrum
3. The nature of the attractor
   a. Local scalings and the trajectory scaling function
   b. The fractional nature of the attractor
4. The Fourier spectrum of period-doubling models
   a. The spectral recursion formula
   b. Spectral moments and interpolations

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Period Doubling in Iterated Maps

J. Rudnick
Department of Physics
University of California
Santa Cruz, California 95064

1. Critical Phenomena: critical exponents and universal scaling functions
2. The period-doubling sequence: the universal numbers α and δ of Feigenbaum
3. The behavior of the Lyapunov exponent in the immediate vicinity of the transition:
   a. Its gross behavior described by a critical exponent
   b. Its fine structure repeated on smaller and smaller scales
4. The structure of orbits in the highly bifurcated regime: a correlation function with power law behavior at the accumulation point
5. External noise, its critical exponent and a universal scaling function for the Lyapunov exponent in the vicinity of the transition in the presence of noise

6. The power spectrum of the iterated map and its behavior near the accumulation point of the series of period-doubling bifurcations
   a. The bifurcation sequence and the appearance of subharmonics
   b. A scaling relation between subharmonics
7. Tests of scaling in the power spectrum
   a. The Rössler attractor
   b. Experimental results in Rayleigh–Bénard convection
8. The appearance of bands and the reverse bifurcation sequence: the origin of noise in the power spectrum on the chaotic side of the transition
9. Interesting recent work on iterated maps

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Studies of the Transition to Turbulent Convection Using Scattered Laser Light

J. P. Gollub
Department of Physics
Haverford College
Haverford, Pennsylvania 19041

I. The Rayleigh–Bénard Instability (Refs. 1, 8)
  1. Parameters of the problem
  2. Relevant hydrodynamic equations
  3. Nonideal boundary conditions in real experiments
  4. Laser light scattering (Ref. 2)

II. Routes to Turbulence at Small Aspect Ratio (Refs. 2, 5, 10, 11, 13)
  1. Multiplicity of spatial states
  2. Quasiperiodicity and phase locking
  3. Analogy to coupled oscillator systems (Ref. 7)
  4. Attempted visualization of a torus
  5. States with three incommensurate frequencies
  6. Subharmonic bifurcations
  7. Complex spatial structure of the oscillations
  8. Intermittent turbulence
  9. Response to externally imposed noise (Ref. 4)

III. Chaos on a Fluid Surface: Faraday's Crispations (Ref. 6)
  1. A symmetry-breaking oscillatory instability
  2. Transition from few to many degrees of freedom

IV. Large Aspect Ratio Convection Experiments (Refs. 3, 9)
1. Existence of the time-independent regime
2. Spatial structures of steady flows: wall effects, defects, dislocations, nonuniqueness, and "annealing"
3. Doppler imaging of the onset of time dependence: a structural instability
4. Statistics of the time dependence above the onset of turbulence: extremely long-lasting correlations
5. Relationship to stability theory and summary

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Experiments on Horizontal Layers of Fluid Heated from Below

G. Ahlers
Department of Physics
University of California
Santa Barbara, California 93106

I. Rayleigh–Bénard Instability
   1. Rounding near convective onset (Refs. 1, 2, 4, 11, 17, 21, 22, 27)
   2. Initial slope of the Nusselt number (Refs. 1, 2, 4, 23, 11, 28, 15, 8)
   3. Time dependence of the evolution of flow (Refs. 10, 8)
   4. Uniqueness of the state above R_c (Refs. 9, 1, 11)
   5. Long-lived transients (Refs. 9, 11)
   6. Non-Boussinesq effects (Refs. 3, 29, 13)

II. Evolution of Turbulence
   1. Small aspect ratios
      a. Limit cycle → torus → strange attractor (Refs. 5, 6, 4)
      b. Period doubling (Refs. 24, 25, 19)
      c. Frequency locking [Refs. 24, 25, Ahlers (unpublished)]
   2. Larger aspect ratio and evidence for stochastic origin of turbulence
      a. Algebraically decaying spectra (Refs. 1, 4, 5, 20)
      b. Oscillatory instability (Refs. 5, 16, 14)
      c. Existence of nonperiodic states immediately above R_c (Refs. 5, 9, 20)
      d. Exponential divergence of the time scale near R_c (Refs. 9, 20)
      e. Large aspect ratio Taylor vortex flow (Refs. 18, 7)

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Instabilities and Chaos in the Couette–Taylor System

Harry Swinney
Department of Physics
University of Texas
Austin, Texas 78712
(in collaboration with M. Gorman, L. A. Reith, and C. D. Andereck)

1. The primary instability in the flow between concentric rotating cylinders (Couette flow)
   a. Rayleigh criterion
   b. Taylor instability
2. Instabilities with the inner cylinder rotating and the outer cylinder at rest
   a. Periodic states (wavy vortex flow): nonuniqueness, dislocations, wave speed variation
   b. Doubly periodic states (modulated wavy vortex flow): space-time symmetries, the rotating annulus
   c. Onset of chaos
   d. Higher instabilities
   e. Other phenomena
3. Instabilities with both cylinders rotating
   a. Counterrotating cylinders
   b. Corotating cylinders

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References 1–8 describe (primarily) the Reynolds number dependence of flows between concentric cylinders with the outer cylinder at rest, in systems with a ratio of cylinder radii of about 0.88 and an aspect ratio of 20 or larger. The dependence of the dynamical behavior on aspect ratio, radius ratio, and outer cylinder speed has scarcely been explored; see, however, Refs. 7, 9, 10, and 11.

1. R. C. DiPrima and H. L. Swinney, "Instabilities in Flow between Concentric Rotating Cylinders," *Hydrodynamic Instabilities and the Transition to Turbulence*, eds. H. L. Swinney and J. P. Gollub (Springer, Berlin, 1981), p. 139. (A review of work through 1980.)
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visualization study that revealed many axial and azimuthal modes in periodic wavy vortex flow at a given Reynolds number.)
4. M. Gorman, H. L. Swinney, and D. Rand, "Doubly Periodic Circular Couette Flow: Experiments Compared with Predictions from Dynamics and Symmetry," *Phys. Rev. Lett.* **46**:992 (1981). (A brief report on the work in Refs. 5 and 6.)
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6. D. Rand, "The Pre-Turbulent Transitions and Flows of a Viscous Fluid between Concentric Rotating Cylinders," *Arch. Rat. Mech. Anal.*, to appear (1981). (Dynamical systems concepts and symmetry are used to derive selection rules for the allowed doubly periodic flows in circularly symmetric systems.)
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Complex Dynamics in a Nonequilibrium Chemical Reaction

Harry Swinney
Department of Physics
University of Texas
Austin, Texas 78712
(in collaboration with J.-C. Roux, J. S. Turner, and W. D. McCormick)

1. The Belousov–Zhabotinskii reaction in a stirred flow reactor
2. Experimental observations of alternating periodic and chaotic regimes in the Belousov–Zhabotinskii reaction
   a. The transition sequence (time series and power spectra)
   b. Deterministic chaos (phase space portraits and return maps)
   c. Comparison of the sequences observed by Hudson and co-workers, the Bordeaux group, and the Texas group

3. Alternating periodic and chaotic regimes in the BZ reactions: a numerical study by J. S. Turner of a Field–Noyes model of the Belousov–Zhabotinskii reaction

References

Until recent years experiments on oscillating chemical reactions concerned closed systems: the chemicals were poured into a closed container and the reaction was observed as the system evolved towards thermodynamic equilibrium. Recent work has shifted to open systems called CSTRs (continuously stirred tank reactors): the chemicals are injected continuously into the vessel and the mixture (of reaction products and unreacted feed) is withdrawn at the same rate. The advantage of flow systems is that the system can be maintained at a well-defined distance away from thermodynamic equilibrium. By far the most extensively investigated and best understood oscillating chemical system is the Belousov–Zhabotinskii (BZ) reaction. Belousov discovered (1959) that this reaction oscillates, and Zhabotinskii (1964) observed spatial as well as temporal oscillations. Subsequent experiments on chemical systems have revealed multiple steady states, simple and complex oscillations, intermittency, chaotic behavior, and complex sequences of instabilities. The following references describe the dynamics of homogeneous (i.e., well-stirred) reactions where diffusion processes can be neglected; inhomogeneous chemical systems are discussed in the lectures by Ortoleva and Segel.

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of alternating chaotic and periodic regimes is found, similar to the one described in Refs. 6 and 7.) See also: J. S. Turner, J.-C. Roux, W. D. McCormick, and H. L. Swinney, "Alternating Periodic and Chaotic Regimes in a Chemical Reaction: Experiment and Theory," *Phys. Lett.* **85A**:9 (1981); J.-C. Roux and H. L. Swinney, "Topology of Chaos in a Chemical Reaction," in *Nonlinear Phenomena in Chemical Dynamics*, A. Pacault and C. Vidal eds. (Springer, New York, 1981).

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Amplitude Equation for the Description of Convection near Onset

M. C. Cross
Bell Laboratories
Murray Hill, New Jersey 07974

I. Derivation of the Amplitude Equation
   1. Method of multiple scales (Newell and Whitehead)
   2. Projection onto critical modes

II. Simple Solutions and Experimental Illustration
   1. Stationary solutions and instabilities in the laterally infinite region
   2. Dynamics: forced phase diffusion and onset experiments

III. Boundary Effects on Pattern Selection
   1. Existence of a Lyapunov functional
   2. Lateral boundary conditions for the envelope function—solution near a boundary and experimental confirmation
   3. Applications—two-dimensional patterns: wave vector selection
      three-dimensional patterns: "textures"

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DERIVATION OF THE AMPLITUDE EQUATION

Basic References: Free–Free Boundary Conditions:

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4. M. C. Cross, "Derivation of the Amplitude Equation at the Rayleigh–Bénard Instability," *Phys. Fluids* **23**:1727 (1980).

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EFFECTS OF LATERAL BOUNDARIES

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Hydrodynamics of Convection

F. H. Busse
Department of Earth and Space Sciences

University of California
Los Angeles, California 90024

I. Weakly Nonlinear Convection (Refs. 1–10, 35, 12, 16, 18, 15, 22)
1. Introduction to Rayleigh–Bénard convection; definitions
2. Linear theory; orientational degeneracy, pattern degeneracy
3. Small amplitude expansion; reduction of degeneracy by nonlinearity
4. Discussion of roll-, square-, hexagon-pattern convection
5. Stability analysis; roll–hexagon competition; hysteresis effects
6. Extremum principle for existence and stability of steady solutions
7. Wavelength changing instabilities (with short movie)
8. Weakly nonlinear convection in spherical shells

II. Fully Nonlinear Convection (Refs. 11, 14, 17, 20, 23–26)
1. Numerical computations of two-dimensional convection
2. Stability analysis of convection rolls as a function of wave number, Rayleigh number, and Prandtl number
3. Thermal boundary layer instability at high Prandtl number; bimodal convection
4. Transition to oscillatory instability at low Prandtl number
5. Comparison with experiments based on controlled initial conditions
6. Oscillatory bimodal and spoke pattern convection
7. Evolution of turbulent convection (with movie)

III. Convection in a Rotating Layer (Refs. 19, 21, 27, 30, 33, 34)
1. General remarks on convection in rotating systems
2. Weakly nonlinear analysis
3. Kuppers–Lortz instability
4. Numerical analysis of two-dimensional convection; stability properties
5. Time dependence of three-mode problem
6. Interaction of patches of convection rolls
7. Transition to turbulence via the statistical limit cycle route
8. Experimental observations (with movie by Heikes and Busse)

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2. G. Z. Gershuni and E. M. Zhukovitskii, *Convection Stability of Incompressible Fluids*, translated from the Russian by D. Louvish (Keter Publications, Jerusalem, 1976).
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12. F. H. Busse, "Patterns of Convection in Plane Layers and Spherical Shells," in *Pattern Formation by Dynamic Systems and Pattern Recognition*, ed. H. Haken (Springer-Verlag, Berlin, 1979), pp. 56–63.

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13. G. Ahlers, "Effect of Departures from the Oberbeck–Boussinesq Approximation on the Heat Transport of Horizontal Convecting Fluid Layers," *J. Fluid Mech.* **98**:137–148 (1980).
14. F. H. Busse, "On the Stability of Two-Dimensional Convection in a Layer Heated from Below," *J. Math. Phys.* **46**:149–150 (1967).
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Topics in the Theory of Convective Structures

E. D. Siggia
Physics Department
Cornell University
Ithaca, New York 14853

I. Onset of Three-Dimensional Convection
   1. Free-slip horizontal boundary conditions in laterally infinite system (exact analytic expansion and numerical simulations)
      a. Effect of vertical vorticity
      b. Modification of earlier amplitude equation and stability diagram near threshold
      c. Effect of Prandtl number

2. Rigid horizontal boundary conditions in laterally infinite system (semiphenomenological theory)
   a. Justification of earlier amplitude equation
   b. Effect of Prandtl number on stability diagram near threshold
3. Effect of lateral boundaries
   a. Periodic boundary conditions
   b. Realistic boundary conditions

II. Defects in Convective Structures
   1. Numerical simulation
   2. Analytic results from amplitude equation

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4. E. D. Siggia and A. Zippelius, "Pattern Selection in Rayleigh–Bénard Convection near Threshold," *Phys. Rev. Lett.* **47**:835 (1981).
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Kinetics of First-Order Phase Transitions

J. D. Gunton
Physics Department
Temple University
Philadelphia, Pennsylvania 19122

I. Kinetics of Metastable States (Refs. 1–11)
   1. Summary of general nucleation theory
   2. Simple model of a binary fluid
   3. First-order phase transition: Critical droplet and surface wobbles (Refs. 2, 11)
   4. Dynamic prefactor
   5. Comparison with experiment: Near-critical completion theory (Refs. 6–10)

II. Spinodal Decomposition (Refs. 12–20)
1. Linear stability analysis
2. Nonlinear theory (Ref. 13)
3. Lifshitz–Slyozov theory
4. Scaling results
5. Renormalization-group calculation of coarse grained free energy (Refs. 14, 20)
6. Outstanding problems

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KINETICS OF METASTABLE STATES

General References:

1. J. S. Langer, "Statistical Theory of the Decay of Metastable States," *Ann. Phys.* (*N.Y.*) **54**:258 (1969).
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General References:

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Introduction to Stochastic Processes

Christiane Caroli
Groupe de Physique des Solides de l'ENS
Université de Paris VII
75221 Paris, France

I. Langevin, Fokker–Planck, and Master Equations—Heuristic Presentation
   (The nature of the physical problem—slow and fast variables)
   1. The Langevin scheme
      a. The linearized Langevin equation: Physical assumptions; time scales (Refs. 1–5)
      b. Diffusion of a free Brownian particle
      c. The fluctuation–dissipation theorem
      d. The retarded Kubo–Mori extension (Refs. 5, 7)
      e. The nonlinear Langevin equation (Ref. 16)
   2. The Fokker–Planck and master equations
      (Distribution function. Separation of time scales and Markov processes)
      a. Transition probabilities in the weak collision limit; the Fokker–Planck equation (Refs. 1, 2, 3, 6)
      b. The limit of rare collisions; the master equation; equilibrium and detailed balance

   c. Weak and rare collisions; equivalence between Fokker–Planck and master equation; objections to the nonlinear FP equation (Refs. 6, 8, 9)

II. Fluctuations and relaxation in systems described by nonlinear FP equations
(The one-variable equation, stationary solution and free-energy minimization)
   a. Linear forces; exact solution (Refs. 1, 2)
   b. Nonlinear forces; stability and metastability; two types of bifurcations: physical examples
   c. Small fluctuations; driving of fluctuations by average values. An example: enhancement of fluctuations during the sweeping of a hysteresis cycle (Refs. 6, 9, 10)
   d. Exchange of particles between locally stable states: Kramers' time (Refs. 11, 12)
   e. Evolution from the vicinity of a point of instability. "Spinodal decomposition" in a one-variable system: Suzuki's time (Refs. 13–15)
   f. Many variable systems (mostly questions)

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Solidification Patterns

J. S. Langer
Physics Department
Carnegie-Mellon University
Pittsburgh, Pennsylvania 15213

I. Directional Solidification: Bénard Analogs
   1. Mullins–Sekerka instability (Refs. 3–5)
   2. Cellular solidification fronts (Refs. 1, 6–9)
   3. Amplitude equations
   4. Pattern-selection problem

II. Directional Solidification of Eutectics
   1. Steady-state theory (Ref. 10)
   2. Instabilities and fluctuations (Refs. 11, 12)
   3. Nonlinear stochastic models (Refs. 2, 11)

III. Dendrites
   1. Steady-state calculations, experiments (Refs. 13–15)
   2. Stability theory (Refs. 1, 16, 17)
   3. Nonlinear models

References

1. J. S. Langer, "Instabilities and Pattern Formation in Crystal Growth," *Rev. Mod. Phys.* **52**:1 (1980).
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3. W. W. Mullins and R. F. Sekerka, "Morphological Stability of a Particle Growing by Diffusion or Heat Flow," *J. Appl. Phys.* **34**:323 (1963).
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16. J. S. Langer and H. Müller-Krumbhaar, "Theory of Dendritic Growth," (Parts I, II, III), *Acta Met.* **26**:1681; 1689; 1697 (1978).
17. H. Müller-Krumbhaar and J. S. Langer, "Sidebranching Instabilities in a Two-Dimensional Model of Dendritic Solidification," *Acta Met.* **29**:145 (1981).

Nonequilibrium Phenomena in Chemistry and Biology

Peter Ortoleva
Department of Chemistry
Indiana University
Bloomington, Indiana 47405

I. Instabilities and Pattern Formation in Chemistry and Geology
   1. Patterning instability in the uniform sol (Refs. 1–6)
      a. Kinetics of first-order phase transitions and transport
      b. The invariant pattern length
      c. Evolution of stochastic initial data
      d. Electroinfusion solitons and their breakdown
   2. Survey of patterning phenomena in rocks (Refs. 7–9)
      a. Plagioclase feldspar periodic zoning: A Stefan problem in complex melt growth
      b. Agates: Banding and twist correlation in fibrous quartz
      c. Orbicular granites
      d. Iron banding
      e. Metamorphic layering
      f. Stylolites
      g. Other examples

3. Metamorphic layering (Refs. 10–12)
   a. Stress and pressure solution kinetics
   b. Kinetic equations
   c. Equilibrium constant functional and the Curie principle
   d. Instability to pattern formation
   e. Numerical simulation of spontaneous metamorphic patterning
4. Stylolites (Ref. 13)
   a. Occurrences
   b. Porosity feedback
   c. Simple and complex mathematical models

II. Biological and Mathematical Topics
1. Developmental bioelectricity
   a. Survey (Refs. 15–19)
   b. Early Fucus egg development
   c. Mechanisms of electrophysiological self-organization
   d. Quantitative modeling
   e. Electrophysiological stability diagram
   f. Polar and quadrupolar (defect) states
   g. Inverted bifurcation
   h. Inherent asymmetry, applied fields and imperfect bifurcation
2. Mathematical methods
   a. Relating attracting manifolds in ODEs and PDEs of chemical reaction: catastrophe and propagation (Refs. 23, 24)
   b. Limit cycles in ODEs: An organizer of complex spatiotemporal behavior (Refs. 25–28)
   c. Padé approximants in nonlinear PDE problems (Refs. 29, 30)

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CHEMISTRY AND GEOLOGY

Spontaneous Pattern Formation in Aging Sols:

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Metamorphic Layering:

10. P. Cobbold, "Description and Origin of Banded Deformation Structures II," *Can. J. Earth Sci.* **14**:2510 (1977).
11. P.-Y. Rubin, "Theory of Metamorphic Segregation and Related Processes," *Geochem. Cosmodium Acta* **43**:1587 (1979).
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Stylolites:

13. E. Merino and P. Ortoleva, "A Kinetic Theory of Stylolite Formation and Spacing," (preprint) and references cited therein.

Periodic Zoning in Plagioclase Feldspars:

14. D. Feinn, S. Haase, J. Chadam, and P. Ortoleva, "Oscillatory Zoning in Plagioclase Feldspar," *Science* **209**:272 (1980)—(and references cited therein).

BIOLOGICAL TOPICS

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20. R. Quatro, *Ann. Rev. Plant Physiol.* **29**:487 (1978).
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MATHEMATICAL METHODS

Large Amplitude Expansion Methods for PDEs
Shocks, Surface Jumping, Catastrophe, and Matched Composite Methods:

23. P. Ortoleva and J. Ross, "Theory of Propagation of Discontinuities in Kinetic Systems with Multiple Time Scales: Fronts, Front Multiplicity, and Pulses," *J. Chem. Phys.* **63**:3398 (1975).
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ODE Limit Cycles and Spatiotemporal Dynamics:

25. P. Ortoleva and J. Ross, "On a Variety of Wave Phenomena in Chemical Reactions," *J. Chem. Phys.* **60**:5090 (1974).
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Padé Approximants in PDEs:

29. P. Ortoleva, "Dynamic Padé Approximants in the Theory of Periodic and Chaotic Chemical Center Waves," *J. Chem. Phys.* **69**:300 (1978).
30. Sh. Bose, S. Bose, and P. Ortoleva, "Dynamic Padé Approximants for Chemical Center Waves," *J. Chem. Phys.* **72**:4258 (1980).

Nonequilibrium Phenomena in Biology and Ecology

Lee A. Segel
Department of Applied Mathematics
Weizmann Institute
Rehovot, Israel

1. Slime mold aggregation
   a. The Keller–Segel equation (Refs. 1–3)
   b. Simulations (Refs. 4, 5)
   c. Developmental transitions in cAMP signaling (Ref. 6)
2. Chemotactic bacteria: traveling waves and biased random walk (Refs. 7–9)
3. The Turing–Meinhardt–Gierer reaction–diffusion approach to morphogenesis (Refs. 10–12)
4. The mechanics of epithelial folding, invagination, and periodic thickening (Refs. 13, 14)
5. Growth and morphogenesis in fungi
6. Bifurcation in aspect space for predator–prey systems (Ref. 15)

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12. Also see section by L. Segel in M3CB (Ref. 9) on PDEs of morphogenesis.
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14. G. Oster, G. Odell, and P. Alberch, "Mechanics, Morphogenesis and Evolution," *Lect. Math. Life Sci.* **13**:165 (1980).
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